Modeling the Effects of Mergers in Procurement: Addendum

Nathan H. Miller*

Georgetown University

February 19, 2017

This addendum clarifies the notation used in the Miller (2014) model of mergers in procurement. It is assumed that the reader is familiar with the model and notation of that article.

A   Stochastic Properties

The right-hand-side of equations (7)-(10) in Section 3 of the article are conditional on firm j winning the auction. These equations govern the expected price, buyer utility, and supplier profit. The following equations make this explicit. Expected variable profit is

$$E\left[\pi_j|\ w_{ij} > z_{\{k \neq j\}}; \boldsymbol{\theta}\right] = E\left[z_N|\ w_{ij} > z_{\{k \neq j\}}; \boldsymbol{\theta}\right] - E\left[z_{\{k \neq j\}}|\ w_{ij} > z_{\{k \neq j\}}; \boldsymbol{\theta}\right] \qquad (A.1)$$

The conditional expected buyer utility equals

$$E\left[u_i|\ w_{ij} > z_{\{k \neq j\}}; \boldsymbol{\theta}\right] = E\left[z_{\{k \neq j\}}|\ w_{ij} > z_{\{k \neq j\}}; \boldsymbol{\theta}\right] \qquad (A.2)$$

The conditional expected price conditional equals

$$E\left[p_i|\ w_{ij} > z_{\{k \neq j\}}; \boldsymbol{\theta}\right] = E\left[v_{ij}|w_{ij} > z_{\{k \neq j\}}; \boldsymbol{\theta}\right] - E\left[z_{\{k \neq j\}}|\ w_{ij} > z_{\{k \neq j\}}; \boldsymbol{\theta}\right] \qquad (A.3)$$

*Georgetown University, McDonough School of Business, 37th and O Streets NW, Washington DC 20057. Email: nhm27@georgetown.edu.

Adding and subtracting expected marginal costs, and substituting based on equation (A.1), yields an expression in terms of marginal costs and a markup term:

$$E\left[p_i | \, w_{ij} > z_{\{k \neq j\}}; \boldsymbol{\theta}\right] = E\left[c_{ij} | w_{ij} > z_{\{k \neq j\}}; \boldsymbol{\theta}\right] + E\left[\pi_j | w_{ij} > z_{\{k \neq j\}}; \boldsymbol{\theta}\right]. \tag{A.4}$$

The expressions provided for the merger effects in Section 4 similarly are conditioned on the winning supplier. For brevity, I do not rewrite those equations here.

B The Gumbel Distribution

Closed-form solutions can be obtained for the specific case that surplus has the Gumbel distribution. For instance, the expected markup of supplier j, conditional on winning, can be expressed as

$$
\begin{aligned}
E\left[m_j | \, w_{ij} > z_{\{k \neq j\}}; \boldsymbol{\theta}\right] &= E\left[p_i | \, w_{ij} > z_{\{k \neq j\}}; \boldsymbol{\theta}\right] - E\left[c_{ij} | w_{ij} > z_{\{k \neq j\}}; \boldsymbol{\theta}\right] && \text{(B.1)} \\
&= \frac{1}{s_j} \sigma \log\left(\frac{1}{1 - s_j}\right) && \text{(B.2)}
\end{aligned}
$$

This can be paired with equation (21) or (24) to recover the $J + 1$ parameters of the model from data on market shares and one firm's average markup. Considering a merger between suppliers are j and n, the expected post-merger markups of these suppliers are given by

$$E\left[m_j | \, w_j > z_{\{k \neq j\}}; \boldsymbol{\theta}\right] = E\left[m_n | \, w_n > z_{\{k \neq n\}}; \boldsymbol{\theta}\right] = \frac{1}{s_j + s_n} \sigma \log\left(\frac{1}{1 - s_j - s_n}\right) \tag{B.3}$$

References

Miller, Nathan (2014). "Modeling the Effects of Mergers in Procurement." *International Journal of Industrial Organization* 37 (November): 201-208.